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**SPECIES DIFFUSION IN
THE FROZEN LAMINAR BOUNDARY LAYER
ON A CATALYTIC FLAT PLATE**

R. J. Vidal

Cornell Aeronautical Laboratory, Inc.

Buffalo, New York

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FOREWORD

The research presented herein was sponsored by Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Program Element 61445014, Project 8951, Task 895106.

A program of research is in progress at CAL to develop a catalytic probe as a diagnostic device for use in high temperature test facilities. This program is sponsored by AEDC under Contract No. AF 40(600)-1093, and is under the technical cognizance of Captain C. O. Forsythe. This is an interim technical report describing the results of a theoretical study to provide a better basis for interpreting data obtained with a catalytic probe. The manuscript was submitted for publication on April 18, 1967.

The reproducibles used in the reproduction of this report were supplied by the author.

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This technical report has been reviewed and is approved.

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ABSTRACT

A theoretical solution is presented for species diffusion in the compressible laminar boundary layer on a flat plate for frozen gas-phase chemistry. It is assumed that the recombination at the wall is a first-order reaction, and the chordwise concentration gradients are neglected. The solution parallels Crocco's treatment except for the assumption governing the viscosity variations with temperature. A modified Chapman-Rubesin constant is used here, and it is assumed that the product of the density and viscosity varies as a power of the static enthalpy. This makes it possible to closely approximate the actual $\rho\mu$ -product in the vicinity of the wall.

The solution for the species equation and the energy equation are obtained in terms of the shear in the boundary layer. The latter is determined following an empirical calculation used by Young. The result given here should be somewhat more general since the Prandtl number effects are included in a more complete fashion. The solutions for heat transfer and skin friction are compared with existing exact numerical solutions to show that they agree with typical errors of about 5 percent.

LIST OF SYMBOLS

α	Species mass fraction
C_p	Specific heat at constant pressure
\bar{C}_p	$\sum_i \alpha_i C_{p_i}$
T	Temperature
I	$\bar{C}_p T$
h_{D_i}	Heat of formation for the i^{th} species
h	Static enthalpy of the mixture $I + \sum \alpha_i h_{D_i}$
H_T	Total enthalpy
ρ	Density
μ	Viscosity
D_{12}	Binary diffusion coefficient
k	Thermal conductivity
σ	Prandtl number $\mu \bar{C}_p / k$
Sc	Schmidt number $\mu / \rho D_{12}$
Le	Lewis number σ / Sc
x, y	Cartesian coordinates parallel to and normal to the ambient stream
u, v	Velocity components in the x and y directions
τ	Shear, $\mu \frac{\partial u}{\partial y}$
R	Universal gas constant
m	Atomic weight
γ_c	Catalytic efficiency, the ratio of atoms recombining on the surface to the atoms incident on the surface
k_w	Wall reaction speed $\gamma_c \sqrt{\frac{RT_w}{2\pi m}}$

$$\phi(x) \quad \tau = \phi(x) f(u)$$

$$f(u) \quad \tau = \phi(x) f(u)$$

$$\eta \quad u/u_e$$

$$i \quad I/I_e$$

$$\xi \quad \alpha/\alpha_e$$

$$T(\eta) \quad \text{Equation 9}$$

$$F(\eta) \quad \text{Equation 11}$$

$$\omega \quad \text{The viscosity exponent, } \mu \sim T^\omega$$

$$\omega_{eff} \quad \text{Equation 10}$$

$$C'_* \quad \text{Equation 10}$$

$$q \quad \text{Heat transfer rate per unit area}$$

$$C_f \quad \text{Skin friction coefficient } 2\tau/\rho_e u_e^2$$

$$C_H \quad \text{Stanton number } \frac{q}{\rho_e u_e (H_T - I_w)}$$

$$Re \quad \text{Reynolds number } \rho_e u_e x / \mu_e$$

$$A(\eta, \sigma) \quad \text{Equation 12}$$

$$B(\eta, \sigma) \quad \text{Equation 12}$$

$$E(x, \sigma) \quad \left[\frac{\rho_e u_e}{\rho_w K_w} \right] \left[\frac{C_f \sqrt{Re}}{A_1(I, \sigma)} \right] \frac{1}{2\sigma \sqrt{Re}}$$

$$a_1, a_2, b_1, b_2 \quad \text{Equation 18}$$

Subscripts

$$i \quad i^{th} \text{ species}$$

$$w \quad \text{Wall conditions}$$

$$e \quad \text{Conditions at the edge of the boundary layer}$$

I. INTRODUCTION

A program of research is in progress at CAL aimed at developing a catalytic probe as a diagnostic device for measuring species concentrations in nozzle expansions of high temperature air. This device was originally suggested by Hoenig,¹ and has received increasing attention in recent years. Basically, it consists of two adjacent heat transfer gages. One is coated with a surface which is catalytic to atom recombination while the other is coated with a noncatalytic surface. The difference between the heat transfer rates sensed by the two gages is proportional to the number of atoms striking the surface, and this can be related to the ambient atom concentration if an appropriate theoretical solution is available.

Theoretical studies of catalytic surfaces in reacting flows have been pursued for about a decade and solutions are presently available for species diffusion in the laminar boundary layer both for the frozen case²⁻¹¹ and for the case where there is species production.¹²⁻¹⁴ A variety of configurations are treated in these references including the flat plate with and without pressure gradient, wedges, cones, and blunt bodies. These also cover a range of assumptions for the transport properties including a linear viscosity-temperature relation, constant Prandtl and Schmidt numbers which differ from unity, and variable Prandtl and Schmidt numbers. With this wide assortment of theoretical solutions available, it was expected that experimental data could be interpreted within the framework of at least one of them. Early experimental results showed, however, that there were important effects apparently stemming from the transport parameters (Prandtl number and Schmidt number) as well as variations in the product of the density and viscosity. This might have been anticipated from the numerical solutions given by Janowitz and Libby.⁸ In that work, numerical solutions were obtained for the frozen laminar boundary layer for certain specific cases. Those included constant transport parameters as well as realistic models for the transport processes in the boundary layer. The comparison of the computed results obtained with the usual assumptions for the transport parameters ($\sigma = Sc = 1, \frac{\rho\mu}{\rho_e\mu_e} = \text{constant}$) discloses large and important effects stemming

from these approximations. The effects of Prandtl and Schmidt numbers that differ from unity can be accounted for with Inger's extension⁵ of Lees' theory.¹⁵ However, all of the cited solutions use the assumption, $\frac{\rho\mu}{\rho_e\mu_e} = \text{constant}$. The purpose of the present report is to present an approximate theoretical solution for species diffusion in the compressible laminar boundary layer on a flat plate which accounts for Prandtl and Schmidt numbers that differ from unity and allows for a $\rho\mu$ -product that varies as a power of the static enthalpy.

A powerful approach in treating the species diffusion in the laminar boundary layer is to take advantage of the fact that with the boundary layer assumptions, the species conservation equation is decoupled from the momentum and energy equations if it is assumed that the Lewis number is unity. This makes it possible to extend existing solutions by writing down the species conservation equation and solving it in terms of the solution for either the momentum or the energy equation. That approach is used here and is applied to Crocco's solution for the flat plate.¹⁶ Crocco's solution, as reported¹⁷ and extended¹⁸ by Young, is used here because it covers the cases of interest, and because the results for skin friction and heat transfer are expressible in simple yet accurate semiempirical formulas.

In the succeeding paragraphs, Crocco's solution is extended to include species diffusion and a solution for the species equation in terms of the solution to the momentum equation. Following this, Young's semiempirical generalization of Crocco's results for the skin friction and heat transfer is reviewed and extended to better account for Prandtl number effects. The model for the $\rho\mu$ -variations in the boundary layer is critically examined next to arrive at meaningful method for establishing the free constants. Finally, the present results are compared with exact numerical solutions to assess their accuracy.

II. THEORETICAL DEVELOPMENT

The succeeding derivation follows that of Crocco,^{16,17} and his development is repeated here in the interests of completeness. Since the original theory was for an ideal gas, first it must be shown that the energy equation for a nonreacting dissociated gas can be reduced to the energy equation used by Crocco. Using the energy equation given by Dorrance,¹⁹ the procedure consists of subtracting out the momentum equation and defining a mean specific heat for the gas mixture,

$$\bar{C}_p = \sum_i \alpha_i C_{p_i}$$

where α_i and C_{p_i} are the mass fraction and the specific heat at constant pressure, respectively, for the i^{th} species. The static enthalpy is then

$$h = \sum_i \alpha_i h_i = I + \sum_i \alpha_i h_{p_i}$$

where $I = \bar{C}_p \tau$ and h_{p_i} is the heat of formation for the i^{th} species. Using this in the above form of the energy equation, the final form of the energy equation is obtained after subtracting out the species conservation equation. Assuming zero pressure gradient, a constant Prandtl number, σ , and a unit Lewis number, the system of governing relations is

$$\text{momentum} \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad (1)$$

$$\text{continuity} \quad \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (2)$$

$$\text{energy} \quad \rho u \frac{\partial I}{\partial x} + \rho v \frac{\partial I}{\partial y} = \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{1}{\sigma} \frac{\partial}{\partial y} \left(\mu \frac{\partial I}{\partial y} \right) \quad (3)$$

$$\text{species} \quad \rho u \frac{\partial \alpha_i}{\partial x} + \rho v \frac{\partial \alpha_i}{\partial y} = \frac{1}{Sc} \frac{\partial}{\partial y} \left(\mu \frac{\partial \alpha_i}{\partial y} \right) \quad (4)$$

with the boundary conditions

$$\text{for } y = 0: \quad u = 0, \quad I = I_w = \bar{C}_{pw} T_w, \quad \alpha_i(0, x) = \frac{\mu_w}{\rho_w k_w (Sc)_w} \left(\frac{\partial \alpha_i}{\partial y} \right)_w$$

$$\text{for } y = \infty: \quad u \rightarrow u_e, \quad I \rightarrow I_e = \bar{C}_{pe} T_e, \quad \alpha_i \rightarrow \alpha_{ie}$$

Equations 1 through 3 and the accompanying boundary conditions are identically those treated by Crocco. Equation 4 is the additional equation to be solved subject to the indicated first-order wall boundary condition.

Following the Crocco method of solution, the independent variables are changed from (x, y) to (x, u) and a new dependent variable, $\tau = \mu \frac{\partial u}{\partial y}$, is introduced. Making this transformation and then combining the resulting momentum and continuity equations, the transformed system of equations is

$$\text{momentum and continuity} \quad \frac{\partial^2 \tau}{\partial u^2} + u \frac{\partial}{\partial x} \left[\frac{\rho \mu}{\tau} \right] = 0 \quad (5a)$$

$$\text{energy} \quad [1 - \sigma] \tau \frac{\partial \tau}{\partial u} \frac{\partial I}{\partial u} + \tau^2 \left[\sigma + \frac{\partial^2 I}{\partial u^2} \right] - \sigma \rho \mu u \frac{\partial I}{\partial x} = 0 \quad (6a)$$

$$\text{species} \quad \tau^2 \frac{\partial^2 \alpha_i}{\partial u^2} + [1 - S_c] \tau \frac{\partial \tau}{\partial u} \frac{\partial \alpha_i}{\partial u} - S_c \rho \mu u \frac{\partial \alpha_i}{\partial x} = 0 \quad (7a)$$

with the boundary conditions

$$u = 0: \quad I = I_w, \quad \tau = \tau_w, \quad \alpha_i(0, x) = \frac{\tau_w}{\rho_w k_w (Sc)_w} \left(\frac{\partial \alpha_i}{\partial u} \right)_{u=0}$$

$$u = u_e: \quad I = I_e, \quad \tau = 0, \quad \alpha_i = \alpha_{i_e}$$

Crocco's analysis anticipates similar velocity profiles and enthalpy profiles, and therefore, neglects the chordwise variations in these quantities. This step is consistent with the zero pressure gradient assumption if it is assumed that the wall temperature is invariant. It will be assumed here that the chordwise variations in species concentrations are also negligible. This is consistent with the other assumptions and is valid if the wall concentrations are small compared with those in the ambient stream (the diffusion-controlled regime), or if the wall concentration approaches the ambient concentration (the rate-controlled regime). These regimes will be defined in a more quantitative manner after obtaining the solution. Anticipating similar profiles, it is assumed that the shear is of the form

$$\tau = \phi(x) f(u)$$

which when substituted in Equation 5a yield $\phi = \frac{1}{\sqrt{2x}}$. Equation 5a through 7a become

$$f \frac{d^2 f}{du^2} + \rho \mu u = 0 \quad (5b)$$

$$\left[\frac{d^2 I}{du^2} + \sigma \right] f + [1 - \sigma] \frac{df}{du} \frac{dI}{du} = 0 \quad (6b)$$

$$f \frac{d^2 \alpha_i}{du^2} + [1 - S_c] \frac{d\alpha_i}{du} \frac{df}{du} = 0 \quad (7b)$$

Prior to solving the above relations, they are made dimensionless by dividing by the quantities at the edge of the boundary layer

$$u = u_e \eta \quad I = I_e i \quad \alpha_i = \alpha_{ie} \xi \quad (8)$$

and by introducing the new variable

$$\eta(\eta) = f(u) \sqrt{\frac{2}{\rho_e \mu_e u_e^3}} \quad (9)$$

Until now, we have duplicated Crocco's analysis in every detail. At this point it is necessary to depart from that procedure in one respect. If the relations in Equation 8 are substituted in Equation 5b, the second term becomes $\left[\frac{\rho \mu}{\rho_e \mu_e} \right] \eta$, and Crocco then makes the assumption $\mu \sim T^\omega$, so that for an ideal gas, the momentum equation becomes

$$\eta \frac{d^2 f}{d\eta^2} + 2\eta (i)^{\omega-1} = 0$$

It is not possible to proceed as directly in the present problem because the species concentrations vary in the boundary layer and consequently, influence the viscosity and the molecular weight. Instead, it is observed that the power-law assumption for the temperature-viscosity relation is made primarily for the purposes of reducing the $\rho\mu$ -product to a simple function of the enthalpy. Consequently, we will proceed directly to an equivalent assumption.

$$\frac{\rho \mu}{\rho_e \mu_e} = C_*' [i]^{\omega_{eff}-1} \quad (10)$$

and will defer a critical appraisal of this assumption until a later section. It should be noted, however, that C'_* is a constant analogous to the Chapman-Rubens constant²⁰ or to Cheng's improvement on that constant.²¹ Using the dimensionless quantities in Equation 8, the assumption given by Equation 10, and the dimensionless variable,

$$F(\eta) = f(u) \sqrt{\frac{2}{\rho_e \mu_e u_e^3 C'_*}} \quad (11)$$

the final equations are as follows

$$F \frac{d^2 F}{d\eta^2} + 2\eta [i] \omega_{eff}^{-1} = 0 \quad (5c)$$

$$F \left[\frac{d^2 i}{d\eta^2} + \sigma \frac{u_e^2}{I_e} \right] + [1-\sigma] \frac{di}{d\eta} \frac{dF}{d\eta} = 0 \quad (6c)$$

$$F \frac{d^2 \xi}{d\eta^2} + [1-S_c] \frac{dF}{d\eta} \frac{d\xi}{d\eta} = 0 \quad (7c)$$

with the boundary conditions

$$\eta = 0: F = F(0) = \frac{C_f \sqrt{Re}}{\sqrt{C'_*}}, \quad i = i(0) = \frac{\bar{C}_{pw} T_w}{\bar{C}_{pe} T_e}, \quad \xi = \xi(0) = \frac{\tau_w}{\rho_w k_w S_{cw} u_e} \left(\frac{\partial \xi}{\partial \eta} \right)_{\eta=0}$$

$$\eta = \infty: F = 0, \quad i = 1, \quad T = 1$$

Equations 5c and 6c are identical to Crocco's governing equations and his solutions apply directly if it is noted that the nondimensional shear parameter, Equation 11, differs from that used by Crocco. Crocco numerically integrated Equation 5c to obtain solutions to the momentum equation. Both the energy equation and the species equation can be integrated in terms of the non-dimensional shear. The solution for the enthalpy distribution is that given in Reference 17

$$i(\eta) = i(0) + \left[\frac{di}{d\eta} \right]_{\eta=0} A_1(\eta, \sigma) - \sigma \frac{u_e^2}{I_e} B_1(\eta, \sigma) \quad (12)$$

where

$$A_1(\eta, \sigma) = \int_0^\eta \left[\frac{F(\eta_1)}{F(0)} \right]^{\sigma-1} d\eta_1, \quad B_1(\eta, \sigma) = \int_0^\eta \left[\frac{F(\eta_1)}{F(0)} \right]^{\sigma-1} \left\{ \int_0^{\eta_1} \left[\frac{F(\eta_2)}{F(0)} \right]^{1-\sigma} d\eta_2 \right\} d\eta_1$$

are tabulated in Reference 17 as a function of η and σ . The values of these quantities evaluated at the edge of the boundary layer enter into the solution and are accurately represented (errors of less than 2 percent) by

$$A_1(1, \sigma) = \sigma^{-1/3} \quad 2B_1(1, \sigma) = \sigma^{-1/2}$$

The species conservation equation, Equation 7c, can be integrated immediately, and after applying the boundary conditions, the solution is

$$\begin{aligned} \left(\frac{d\xi}{d\eta} \right)_{\eta=0} &= \frac{\left[\frac{1}{A_1(1, \sigma)} \right]}{1 + E(x, \sigma)} \\ \xi &= \frac{E(x, \sigma)}{1 + E(x, \sigma)} + \frac{\left[\frac{A_1(\eta, \sigma)}{A_1(1, \sigma)} \right]}{1 + E(x, \sigma)} \end{aligned} \quad (13)$$

where

$$E(x, \sigma) = \left[\frac{\rho_e u_e}{\rho_w k_w} \right] \left[\frac{C_f \sqrt{Re}}{A_1(1, \sigma)} \right] \frac{1}{2\sigma \sqrt{Re}} = \left[\frac{\rho_e u_e}{\rho_w k_w} \right] \left[\frac{C_f \sqrt{Re}}{2\sigma^{2/3}} \right] \frac{1}{\sqrt{Re}}$$

The similarity assumption, neglecting the chordwise variations in species concentrations, can now be assessed. Since the present analysis is restricted to zero pressure gradient, the only chordwise variations must result from variations in the wall concentrations. Equation 13 shows that the wall concentration is

$$\xi_w = \frac{E(x, \sigma)}{1 + E(x, \sigma)} = \begin{cases} 0 & \text{for } E(x, \sigma) \rightarrow 0 \\ 1 & \text{for } E(x, \sigma) \rightarrow \infty \end{cases}$$

Consequently, the similarity assumption is valid in the diffusion-controlled regime where $E(x, \sigma)$ is vanishingly small, or equivalently, at large Reynolds numbers. The assumption is also valid in the rate-controlled regime, where the wall reaction speed, k_w , is vanishingly small, or equivalently, where the catalytic efficiency, γ_c , approaches zero.

III. THE SURFACE HEAT TRANSFER AND SKIN FRICTION

The heat transfer to a surface in a gas with a single reacting species is given by Dorrance¹⁹ as

$$-q = \left[\frac{k}{\bar{c}_p} \frac{\partial I}{\partial y} + \rho D_{12} h_o \frac{\partial \alpha}{\partial y} \right]_{y=0} \quad (14)$$

Defining a Stanton number as $C_H = \frac{-q}{\rho_e u_e (H_T - I_w)}$ where $H_T = I_e + \frac{1}{2} u_e^2 + \alpha_e h_o$ and using the solutions as given by Equations 12 and 13, the heat transfer to the wall ($\mathcal{L}_e = 1$) can be expressed in terms of the skin friction coefficient as follows.

$$C_H \sqrt{Re^*} = \frac{C_f \sqrt{Re^*}}{2(\sigma)^{2/3}} \left\{ 1 + \frac{u_e^2/2}{H_T - I_w} [\sqrt{\sigma^2 - 1}] - \frac{\alpha_e h_o}{H_T - I_w} \xi_w \right\} \quad (15)$$

where

$$\xi_w = \frac{E(x, \sigma)}{1 + E(x, \sigma)}$$

The skin friction was determined by Crocco by numerically integrating the momentum equation. Young¹⁸ subsequently generalized those results in a semiempirical calculation to obtain a closed-form expression for the skin friction. Young's calculation will be extended here to better account for the influence of the Prandtl number. The basis for the calculation follows from Equation 5c which can be put in the following form

$$F(0) \equiv C_f \sqrt{\frac{Re^*}{c_*}} = \int_0^1 \left\{ \int_0^{\eta_1} \frac{2\eta [i]}{F(\eta)} \omega_{eff}^{-1} d\eta \right\} d\eta_1 \quad (16)$$

Young notes that Crocco's numerical results for $F(\eta)$ are closely approximated by $F(0)\sqrt{1-\eta^2}$ which, substituted in Equation 16 yields an approximate expression for the skin friction coefficient.

$$[F(0)]^2 \approx \int_0^1 \left\{ \int_0^{\eta_1} \frac{2\eta}{\sqrt{1-\eta^2}} [i(\eta)] \omega_{eff}^{-1} d\eta \right\} d\eta_1 \quad (17)$$

Equation 17 can be integrated if the enthalpy distribution in the boundary layer is known. Young uses the enthalpy for a unit Prandtl number and introduces a Prandtl number dependence after completing the integration by adjusting the final expression to agree with known exact solutions. The approach used here is to retain Young's expression for the shear distribution in the boundary layer, but to fit a power series to Crocco's solution for the enthalpy distribution, or equivalently, a power series for $A_1(\eta, \sigma)$ and $B_1(\eta, \sigma)$. The expressions used are

$$\begin{aligned} A_1(\eta, \sigma) &= \eta + a_1(\sigma)\eta^4 + a_2(\sigma)\eta^8 \\ B_1(\eta, \sigma) &= \frac{1}{2} \left[\eta^2 + b_1(\sigma)\eta^5 + b_2(\sigma)\eta^{20} \right] \end{aligned} \quad (18)$$

The parameters, a_1 , a_2 , b_1 , and b_2 are plotted as a function of the Prandtl number in Figure 1. The worst errors between the results tabulated in Reference 17 and Equation 18 were for a Prandtl number of 1/2. Those errors were about 2 to 3 percent and occurred near the edge of the boundary layer.

Equation 17 has been integrated using Equations 12 and 18 for the enthalpy distribution. The procedure for this integration was identical to that used by Young; namely, to note that the expression for the enthalpy is of the following form

$$i \approx C_1 \left\{ 1 + \frac{C_2}{C_1} + \frac{C_3}{C_1} \left[\eta + a_1 \eta^4 + a_2 \eta^8 \right] + \frac{C_4}{C_1} \left[\eta^2 + b_1 \eta^5 + b_2 \eta^{20} \right] \right\}$$

and that each of the quantities in the brace are always less than unity. This suggests that the enthalpy function can be approximated by the first term in the series expansion,

$$\begin{aligned} i^{\omega_{eff}^{-1}} &\approx C_1^{\omega_{eff}^{-1}} \left\{ 1 + \left[\omega_{eff}^{-1} \right] \frac{C_2}{C_1} + \frac{C_3}{C_1} \left[\omega_{eff}^{-1} \right] \left[\eta + a_1 \eta^4 + a_2 \eta^8 \right] \right. \\ &\quad \left. + \frac{C_4}{C_1} \left[\omega_{eff}^{-1} \right] \left[\eta^2 + b_1 \eta^5 + b_2 \eta^{20} \right] + \dots \right\} \end{aligned}$$

After the integration has been completed, the procedure is reversed to yield the original power dependence. The details of this procedure can be obtained in Reference 18. The integration and subsequent return to the original form yields the following result

$$F(0) \equiv C_f \sqrt{\frac{Re}{C_*'}} \approx \sqrt{2 \left[1 - \frac{\pi}{4} \right]} \left\{ C_1 + C_2 + \left[\frac{C_3}{1 - \frac{\pi}{4}} \right] \left[0.1187 + 0.0424 a_1(\sigma) - 0.2094 a_2(\sigma) \right] + \left[\frac{C_4}{1 - \frac{\pi}{4}} \right] \left[0.0777 + 0.1379 b_1(\sigma) - 0.0103 b_2(\sigma) \right] \right\} \left[\frac{\omega_{eff} - 1}{2} \right]$$

It is obvious that this procedure is approximate and can only be justified by comparing the result with exact solutions. As shown by Young,¹⁸ it does however yield a result in close agreement with the Blasius limit. For the case where $\omega_{eff} \approx 1$, $F(0) = C_f \sqrt{Re} = 0.655$ (since $C_*' = 1$) as compared with 0.664 for the Blasius limit. This close agreement suggests the final form for the skin friction coefficient.

$$C_f \sqrt{Re} = 0.664 \sqrt{C_*'} \left\{ K_1(\sigma) + [1 - K_1(\sigma)] i(0) + K_2(\sigma) \left[\frac{u_e^2}{2 I_e} \right] \right\} \left[\frac{\omega_{eff} - 1}{2} \right] \quad (19a)$$

The constants, $K_1(\sigma)$ and $K_2(\sigma)$, are related to the constants a_1 , a_2 , b_1 , and b_2 and are functions only of the Prandtl number. They are plotted in Figure 2 and compared with the values recommended by Young. It can be seen that they are in reasonable agreement at $\sigma = 1$, but the parameters arrived at here show a rather strong Prandtl number dependence. They can be approximated by the following relations

$$K_1(\sigma) \approx 0.55 \sigma^{2/3} \quad K_2(\sigma) \approx 0.0964 \left[\sigma^{-5/3} + \sigma^{1/3} \right]$$

to yield for the skin friction

$$C_f \sqrt{Re} = 0.664 \sqrt{C_*'} \left\{ i(0) + 0.55 \sigma^{2/3} [1 - i(0)] + 0.0964 \left[\sigma^{-5/3} + \sigma^{1/3} \right] \frac{u_e^2}{2 I_e} \right\} \left[\frac{\omega_{eff} - 1}{2} \right] \quad (19b)$$

Equations 15 and 19 are the final relations for the heat transfer and skin friction coefficients. They both depend on two constants, C_*' and ω_{eff} , which are as yet unspecified. These will be determined in the next section where consideration is given to the significance of these parameters.

IV. DENSITY-VISCOSITY PRODUCT

A basic difficulty in any compressible laminar boundary layer development is in treating the product of the density and the viscosity. There is a variety of approximate techniques used in the theory for nonreacting gases, and it is natural to look to that literature for a guide. The simplest approach in nonreacting gases is to assume that the viscosity is linearly related to the temperature so that within the boundary layer approximations, the $\rho\mu$ -product is constant through the boundary layer. This procedure results in the familiar Chapman-Rubesin constant, which makes it possible to match the viscosity at one point in the flow field. The original choice²⁰ was at the wall, and it was found to be accurate at supersonic speeds. Cheng²¹ refined this procedure by matching the viscosity at the point in the boundary layer corresponding to the average temperature. This approach has been used at relatively high temperatures (3000°K) and shown to produce accurate results when compared with experimental data.^{21,22} It has been shown to lead to serious errors, however, when applied at temperatures of 5000 to 6000°K.²³ The power-law approximation used by Crocco is useful since it makes it possible to match the viscosity at one point in the flow field and to approximate the actual temperature dependence. A better approximation is to introduce the Chapman-Rubesin constant in conjunction with the power-law approximation as done in Reference 23. This makes it possible to match the viscosity at two points in the flow field while approximating the actual temperature dependence.

None of these approximations to the viscosity is useful in the present problem owing to the complications introduced by the variable species concentrations in the boundary layer. The species concentration changes the molecular weight and thereby introduces a dependence on the concentrations as well as on the temperature. Depending on the model used for estimating the viscosity, the species concentrations may or may not enter into that quantity too. Because of these complications, the approach used here is to relate the $\rho\mu$ -product to the enthalpy using an approximation analogous to that used in approximating just the viscosity.

The approximation introduced in Equation 10 is analogous to that used in Reference 23

$$\frac{\rho\mu}{\rho_e\mu_e} = C'_* [\bar{i}]^{\omega_{eff}-1}$$

except that it must be noted that the exponent, ω_{eff} , is not necessarily the exponent normally used in relating the viscosity to the temperature. It and C'_* are free constants to be determined by matching the actual $\rho\mu$ -product at two points in the boundary layer. The two points chosen here are at the wall and at the point in the boundary layer corresponding to the average value of \bar{i} . This procedure is analogous to that used in References 21 and 23, where the average temperature is used to determine the modified Chapman-Rubesin constant. The second point is chosen at the wall because interest centers on the transport processes at the wall.

The average value of the enthalpy, \bar{i} , can be determined by integrating Equation 12, using Equation 18 to approximate the Prandtl number dependence. The result is

$$\bar{i} = i(0) + K_3(\sigma) [1 - i(0)] + K_4(\sigma) \frac{u_e^2}{2I_e} \quad (20a)$$

The constants $K_3(\sigma)$ and $K_4(\sigma)$ are functions only of the Prandtl number and are plotted in Figure 3. They can be closely approximated by the relations,

$$K_3(\sigma) \approx \frac{1}{2} \sigma^{2/7} \quad K_4(\sigma) \approx \frac{1}{6} \sigma^{3/5}$$

and the relation for the average enthalpy becomes

$$\bar{i} = i(0) + \frac{1}{2} \sigma^{2/7} [1 - i(0)] + \frac{1}{6} \sigma^{3/5} \frac{u_e^2}{2I_e} \quad (20b)$$

The average enthalpy and the species concentration corresponding to the average value of the enthalpy, $\xi(\bar{i})$, are used to determine the $\rho\mu$ -product that corresponds to the average value of the enthalpy. The species concentration is computed from Equation 13 using the value of η corresponding to the average enthalpy, $\eta(\bar{i})$. For unit Prandtl number, this value can be determined from the quadratic relation,

$$\left[\eta(\bar{i}) \right]^2 - \left[\eta(i) \right] \left[1 + \frac{1 - i(0)}{u_e^2/2I_e} \right] + K_4(\sigma) + K_3(\sigma) \left[\frac{1 - i(0)}{u_e^2/2I_e} \right] = 0 \quad (21)$$

For Prandtl numbers other than unity, the value of $\eta(\bar{i})$ can be determined from Figure 4. The free constants ω_{eff} and C'_* are then given by the following relations.

$$\omega_{eff} = 1 - \frac{\ln \left[\frac{\rho_w \mu_w}{\rho(\bar{i}) \mu(i)} \right]}{\ln \left[\frac{\bar{i}}{i(0)} \right]} \quad (22)$$

$$C'_* = [i(0)]^{1-\omega_{eff}} \frac{\rho_w \mu_w}{\rho_e \mu_e}$$

V. COMPARISON WITH EXACT SOLUTIONS

The present theory has been applied to those cases treated by Janowitz and Libby,⁸ and the results are compared with the exact numerical solutions in Table I. The present theory was evaluated using the gas model and the relations for the transport properties that were used in Reference 8. The only additional computation made here was to calculate the total temperature at the edge of the boundary layer using real gas charts for nitrogen and taking the density altitude as 250,000 feet. The theory of Reference 8 makes the same similarity assumptions used here. Consequently, the comparison sheds no light on the accuracy of that approximation.*

There are four basic flows covered in Table I. The first is one in which the Prandtl and Schmidt numbers are unity and the $\rho\mu$ -ratio is equal to one. This is the Blasius limit, and Table I shows that the present treatment reduces to that limit with good accuracy.

The second basic flow covered in Table I is one in which the $\rho\mu$ -ratio is unity, and the Prandtl and Schmidt numbers are 0.7. These cases provide a check on the methods used to account for the transport parameters in computing the heat transfer, and Table I shows that the solutions agree within 3 percent.

The third flow situation in Table I is one in which the transport parameters are unity, but the $\rho\mu$ -ratio varies through the boundary layer. This flow provides an excellent test of the present theory since it closely fits the theoretical assumptions ($\mathcal{L} = 1$, $S_c = \sigma = \text{constant}$). Consequently, the comparison will furnish a good check on the method used to account for the $\rho\mu$ -variation in the boundary layer. The calculations for this case followed the procedures outlined in the previous sections with one addition.

*The accuracy of the similarity assumption can be assessed from the results given by Chambré and Acrivos.² They test it over the complete range, from zero to infinite catalycity, and show that the largest error in wall concentration is about 10 percent occurring at values of $E(\chi, \sigma)$ of about 1/2.

In determining the value of $\rho\mu$ corresponding to the average enthalpy, \bar{h} , the quadratic relation for $\eta(\bar{h})$, Equation 21, yielded two values. This is due to the fact that the wall temperature and the temperature at the edge of the boundary layer are nearly equal. The value used was that closest to the wall. This procedure is consistent with the objective of matching the $\rho\mu$ -ratio in the vicinity of the wall. The comparison with the exact solutions in Table I shows that the present theory predicts the skin friction with errors of less than 5 percent. The two solutions for heat transfer show typical discrepancies of 3 to 4 percent, though in one instance (Case 2) the discrepancy is 7 percent. In general, it appears that the theory is quite accurate when applied to these cases.

The fourth situation covered in Table I closely corresponds to an actual flow in that the transport parameters and the $\rho\mu$ -ratio vary through the boundary layer. The calculations for this case should follow an iteration procedure to determine the atom concentrations and the transport parameters at the wall. Instead, the transport parameters and the concentrations at the wall given by Janowitz and Libby were used to calculate the flow properties used in determining C'_* and ω_{eff} (Equation 22). With these so determined, the skin friction, heat transfer, atom concentration, and $\rho\mu$ -ratio at the wall were calculated. Table I shows that the discrepancies between the two solutions for skin friction are typically 5 percent and are always less than 10 percent. The difference between the two solutions for heat transfer are typically 5 percent, though in one instance (Case 8) the discrepancy is 14 percent and in one other (Case 2) it is 10 percent. These two errors appear to stem largely from the assumption that the Lewis number is unity and can be partially accounted for by using the complete expression for heat transfer.

$$C_H \sqrt{Re} = \frac{C_f \sqrt{Re'}}{2(\sigma_w')^{2/3}} \left\{ 1 + \frac{u_e^2/2}{H_T - I_w} \left[\sqrt{\sigma_w'} - 1 \right] - \frac{\alpha_e h_D}{H_T - I_w} \left[\xi_w + \frac{1 - \xi_e}{1 + E(\chi, \sigma')} \right] \right\}$$

If the above expression is used to compute the heat transfer for these two cases, the discrepancies are less than 5 percent. There is, of course, no firm justification for this procedure and it can only be viewed as indicative of the source of the discrepancies.

VI. CONCLUDING REMARKS

Crocco's theory for the compressible laminar boundary layer on a flat plate has been extended to obtain an approximate analytic boundary layer solution for species diffusion to a catalytic surface. This extension has paralleled the development as reported by Young except for the viscosity assumption and the empirical treatment for the skin friction. Instead of using Crocco's assumption that the viscosity varies as a power of the temperature, the assumption made here was that the product of the density and viscosity varied as a power of the static enthalpy ($\bar{c}_p \tau$). In addition, a modified Chapman-Rubesin constant is introduced to improve the accuracy of the assumption. This procedure is in the spirit of the usual viscosity assumption except that it lumps together the viscosity variations and species variations, as they influence the $\rho\mu$ -ratio.

The empirical treatment of the skin friction given here differs from that given by Young in that his is based on the enthalpy profile for unit Prandtl number. The treatment given here attempts to better account for the Prandtl number variations by fitting a power series to the enthalpy profiles.

Solutions obtained with the present theory have been compared with exact numerical solutions to show that typically they agree within about 5 percent. This agreement is regarded as satisfactory since the theory accounts for variations in the $\rho\mu$ -product and since it provides a relatively rapid method for making estimates of the skin friction and heat transfer.

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TABLE I
COMPARISON WITH EXACT NUMERICAL SOLUTIONS

$\sigma = S_c = 1.0$ $\frac{P_w}{P_e} \frac{\mu_w}{\mu_e} = 1.0$										$\sigma = S_c = 0.7$ $\frac{P_w}{P_e} \frac{\mu_w}{\mu_e} = 1.0$										$\sigma = S_c = 1.0$									
CASE	U_w (FPS)	T_w (°K)	T_e (°K)	T_{oe} (°K)	σ_c	$\frac{\alpha_w}{\alpha_e} \frac{P_w}{P_e}$	S	$\frac{\alpha_w}{\alpha_e}$ ①	$C_F \sqrt{Re_F}$ ①	$\frac{\alpha_w}{\alpha_e}$ ①	$C_H \sqrt{Re_H}$ ①	$\frac{P_w}{P_e} \frac{\mu_w}{\mu_e}$ ①	$C_F \sqrt{Re_F}$ ①	$\frac{\alpha_w}{\alpha_e}$ ①	$C_H \sqrt{Re_H}$ ①	σ_w	S_c	$\frac{P_w}{P_e} \frac{\mu_w}{\mu_e}$ ①	$C_F \sqrt{Re_F}$ ①	$\frac{\alpha_w}{\alpha_e}$ ①	$C_H \sqrt{Re_H}$ ①	σ_w	S_c	$\frac{P_w}{P_e} \frac{\mu_w}{\mu_e}$ ①	$C_F \sqrt{Re_F}$ ①	$\frac{\alpha_w}{\alpha_e}$ ①	$C_H \sqrt{Re_H}$ ①		
1	26,000	300	400	6200	.75	.738	1/5	.700 .702	.185 .146	.675 .675	.182 .181	1.26 1.28	.503 .526	.565 .593	.138 .139	.681 .808	.808	1.25 1.28	.488 525	.538 .592	.147 .162	.681 .808	.808	1.25 1.28	.488 525	.538 .592	.147 .162		
2	26,000	800	400	6200	.75	.798	1/5	.700 .702	.140 .140	.675 .675	.169 .174	.670 .671	.495 .485	.710 .718	.106 .094	.662 .887	.887	.670 .673	.459 498	.686 .719	.119 .107	.662 .887	.887	.670 .673	.459 498	.686 .719	.119 .107		
3	26,000	300	400	6200	.75	.798	1	.322 .320	.244 .252	.283 .294	.311 .312	1.50 1.59	.539 .542	.147 .135	.234 .332	.757 .629	.629	1.45 1.63	.522 567	.141 .257	.269 .294	.757 .629	.629	1.45 1.63	.522 567	.141 .257	.269 .294		
4	26,000	1000	400	6200	.75	.798	1	.322 .320	.244 .247	.283 .294	.309 .309	.775 .804	.506 .500	.244 .230	.196 .69	.735 .664	.664	.757 .812	.499 .517	.237 .333	.236 .213	.735 .664	.664	.757 .812	.499 .517	.237 .333	.236 .213		
5	26,000	1000	400	6200	.75	.798	10	.044 .045	.320 .324	.043 .040	.396 .397	.905 .944	.541 .516	.025 .026	.264 .254	.785 .589	.589	.640 1.00	.563 532	.022 .148	.317 .316	.785 .589	.589	.640 1.00	.563 532	.022 .148	.317 .316		
6	20,000	300	600	5800	.40	.704	1/5	.700 .702	.165 .170	.675 .675	.198 .204	1.56 1.59	.548 .582	.542 .564	.174 .180	.731 .675	.675	1.55 1.62	.551 .609	.496 .547	.204 223	.731 .675	.675	1.55 1.62	.551 .609	.496 .547	.204 223		
7	20,000	1000	600	5800	.40	.704	1/5	.700 .702	.159 .159	.675 .675	.189 .190					.717 .705	.705	.624 .845	.514 .552	.642 .674	.154 .158	.717 .705	.705	.624 .845	.514 .552	.642 .674	.154 .158		
8	20,000	300	600	5800	.40	.704	1	.322 .320	.256 .264	.283 .294	.316 .323					.723 .606	.606	1.72 1.85	.532 .627	.142 .211	.298 347	.723 .606	.606	1.72 1.85	.532 .627	.142 .211	.298 347		
9	20,000	1000	600	5800	.40	.704	1	.322 .320	.252 .257	.283 .294	.308 .314					.762 .621	.621	.948 .989	.490 .573	.219 .274	.249 .271	.762 .621	.621	.948 .989	.490 .573	.219 .274	.249 .271		

① QUANTITY ON LEFT - PRESENT THEORY
QUANTITY ON RIGHT - JANGWITZ AND LIBBY

THE VALUE OF $N_A = 0.373$ TABULATED BY JANGWITZ AND LIBBY
EVIDENTLY SHOULD BE 0.313.

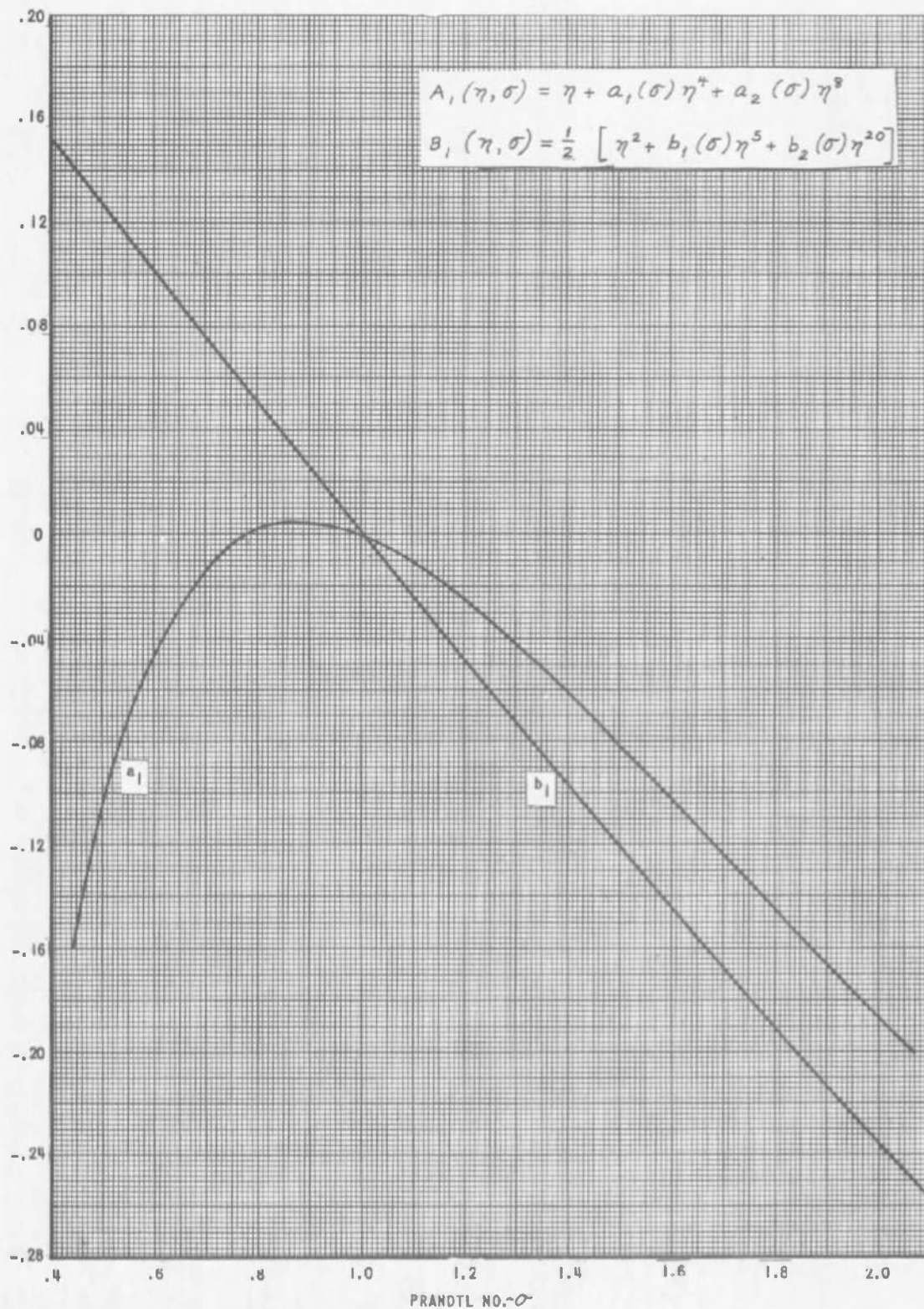


Figure 1 Coefficients in the Solution for the Shear

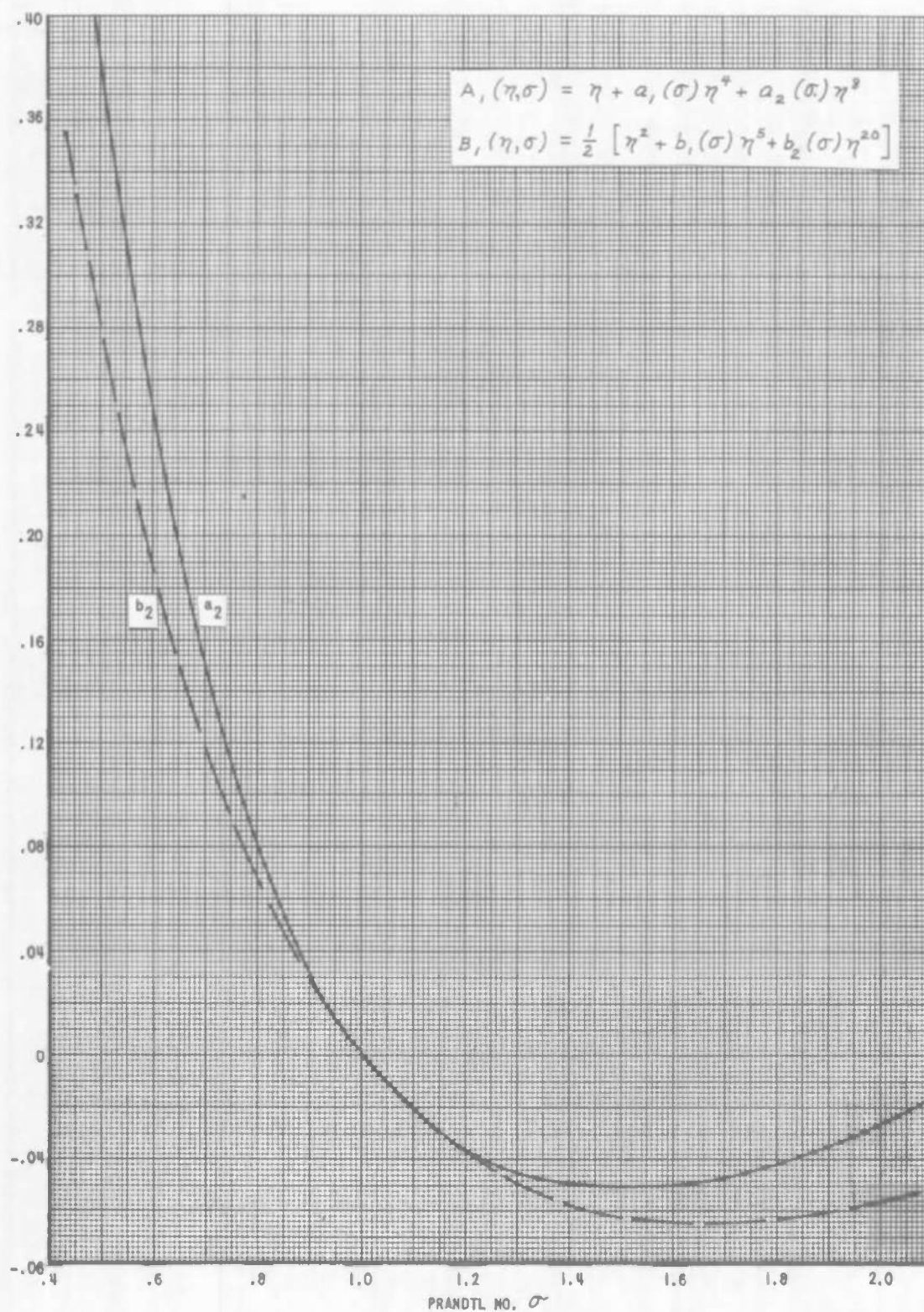


Figure 1 Concluded

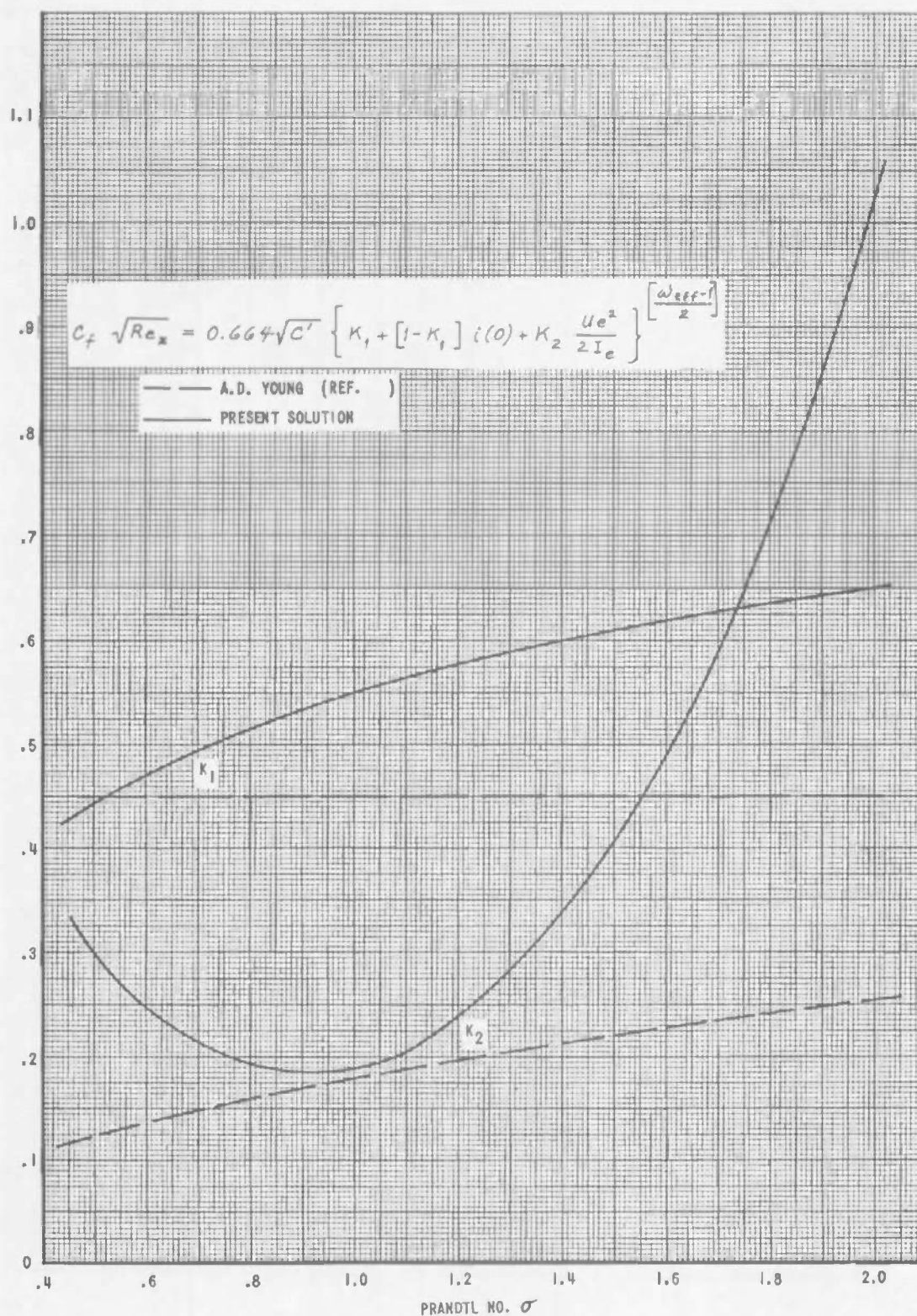


Figure 2 Solution for the Skin Friction

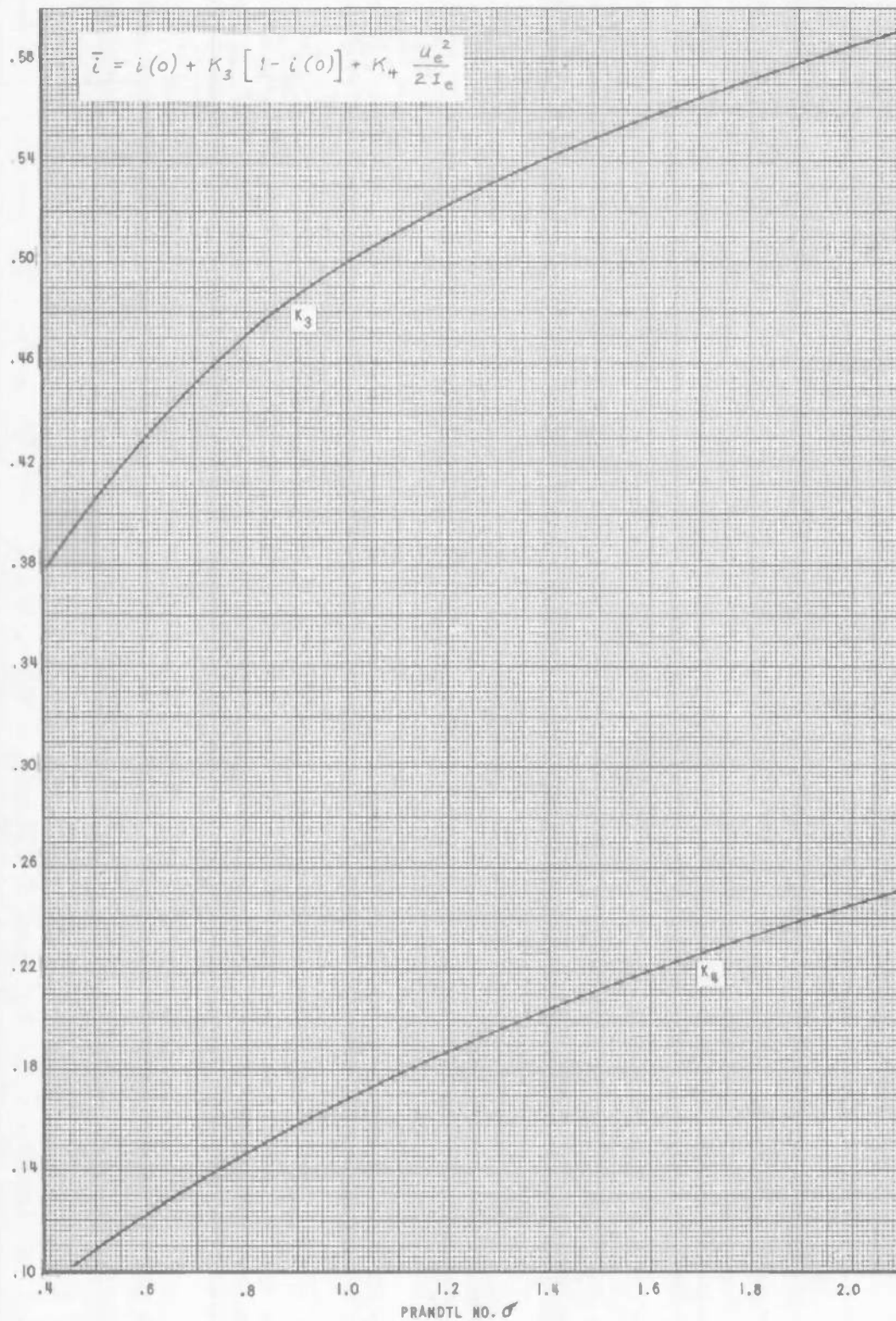


Figure 3 Solution for the Average Enthalpy

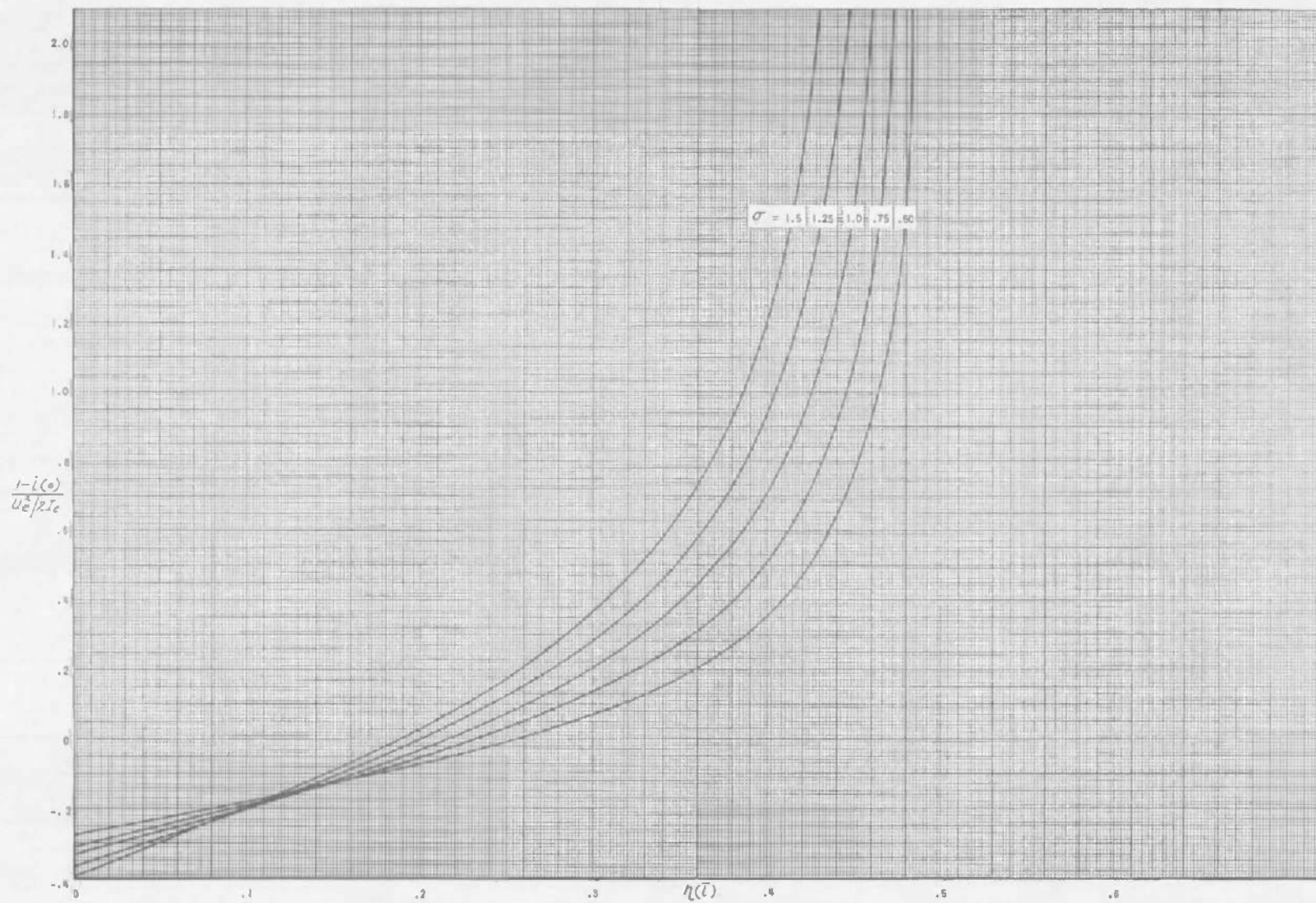


Figure 4 Boundary Layer Coordinate for the Average Enthalpy

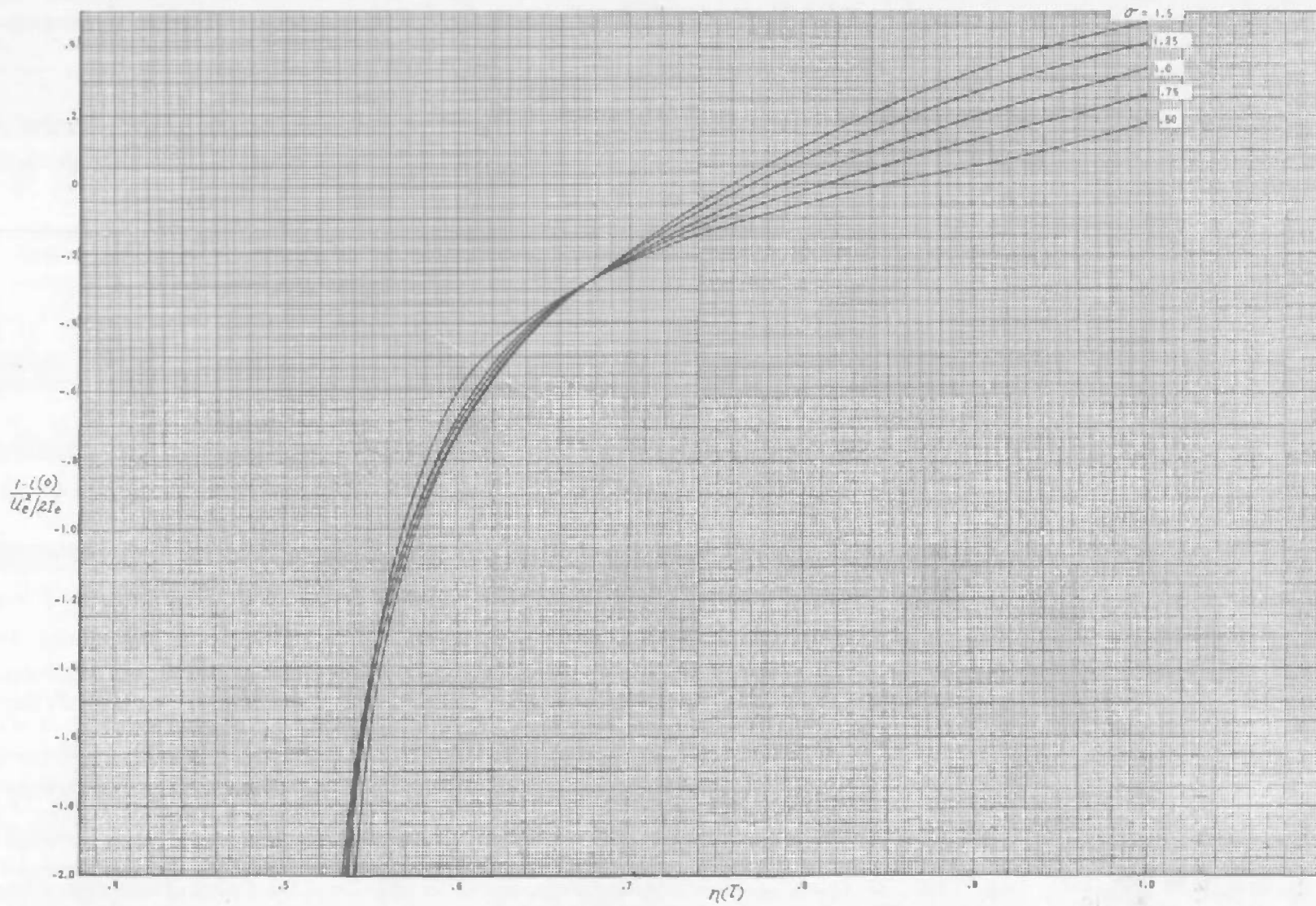


Figure 4 Concluded

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13 ABSTRACT A theoretical solution is presented for species diffusion in the compressible laminar boundary layer on a flat plate for frozen gas-phase chemistry. It is assumed that the recombination at the wall is a first-order reaction, and the chordwise concentration gradients are neglected. The solution parallels Crocco's treatment except for the assumption governing the viscosity variations with temperature. A modified Chapman-Rubesin constant is used here, and it is assumed that the product of the density and viscosity varies as a power of the static enthalpy. This makes it possible to closely approximate the actual $\rho\mu$ -product in the vicinity of the wall. The solution for the species equation and the energy equation are obtained in terms of the shear in the boundary layer. The latter is determined following an empirical calculation used by Young. The result given here should be somewhat more general since the Prandtl number effects are included in a more complete fashion. The solutions for heat transfer and skin friction are compared with existing exact numerical solutions to show that they agree with typical errors of about 5 percent.			

KEY WORDS

LINK A

LINK B

LINK C

ROLE

WT

ROLE

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WT

diffusion

flat plates

2. laminar boundary layer

density

temperature

viscosity

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~~Species Diffusion~~

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